Grade 3 Mathematics, Quarter 4, Unit 4.1

Measuring Mass and Volume

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Measure and estimate liquid volumes using the standard unit of liters.
- Measure and estimate the mass of objects using standard units of grams and kilograms.
- Add, subtract, multiply and divide to solve one-step word problems involving masses or volumes that are given in the same units.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Consider similar problems to gain insight into its solution.

Use appropriate tools strategically.

- Familiar with and able to use tools.
- Ability to decide when tools are appropriate and helpful.
- Using estimation to judge reasonableness of solutions.

Attend to precision**.**

- Define units of measure consistently and appropriately.
- Label for clarification.

Essential questions

- What tools can you use to measure the volume of a liquid?
- What tools can you use to measure the mass of an object?
- What strategies can you use to estimate the volume of a liquid or the mass of an object?
- How can you decide which strategy is best to use to solve a word problem involving mass or volume?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]

[6] Excludes compound units such as cm^3 and finding the geometric volume of a container.

[7] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and

compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2, students measured and estimated lengths using standard units such as ft, cm, and m. They used addition and subtraction within 100 to solve word problems involving lengths that are given in the same units.

Current Learning

Students in grade 3, measure and estimate liquid volumes and masses of objects using the standard units: grams, kilograms and liters. Students add, subtract, multiply or divide to solve one step word problems involving masses or volumes that are given in the same units. This is all at the developmental level.

Routines: Students continue to develop estimation skills in the context of mass and volume.

Future Learning

Grade 4 students will use the four operations to solve word problems involving distances, intervals of time, liquid volumes, mass of objects and money including problems with simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

Additional Findings

Benchmarks for Science Literacy states that "estimation skills can be learned, but only if teachers make sure that students have lots of practice estimating (which happens if estimation is routinely treated as a standard part of problem solving)" (p. 288).

Principles and Standards for School Mathematics states that "Teachers should guide students' experiences by making the resources for measuring available, planning opportunities to measure, and encouraging students to explain the results of their actions. Discourse builds students' conceptual and procedural knowledge of measurement and gives teachers valuable information for reporting progress and planning next steps. The same conversations and questions that help students build vocabulary help teachers learn about students' understandings and misconceptions." (p. 103)

Elementary and Middle School Mathematics: Teaching Developmentally states that "students should have a basic idea of the size of commonly used units and what they measure. It is more important to know approximation rather than having the ability to measure accurately." The book also states that "students need practice in using common sense in the selection of the appropriate standard units" (pp. 387–388) and that "students should know the relationships between units rather than perform tedious conversion exercises which do little to enhance measurement sense" (p. 388).

PARCC Model Content Frameworks Mathematics Grades 3–11 states that, "Continuous measurement quantities such as liquid volume, mass and so on are an important context for fraction arithmetic. In grade 3, students begin to get a feel for continuous measurement quantities and solve whole number problems involving such quantities."

Grade 3 Mathematics, Quarter 4, Unit 4.2

Telling Time and Solving Problems with Time Intervals

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Tell and write time to the nearest minute.
- Measure time intervals in minutes.
- Solve word problems involving addition and subtraction of time intervals in minutes by representing the problem on a number line or diagram.

Mathematical practices to be integrated

Model with mathematics.

- Apply the mathematics they know to solve problems in everyday life.

Attend to precision.

- Label for clarification.
- Define math symbols and units of measure consistently and appropriately.

Essential questions

- How does the hour hand relate to the numbers on the clock? The minute hand?
- If the minute hand is on the 4, how many minutes does this represent?
- Each number on a clock represents how many minutes? How many minutes are there in an hour?
- How is time related to a number line?
- What is the relationship between hours, days, weeks, and years?
- When would someone need to calculate elapsed time?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 2, students told and wrote time from analog and digital clocks to the nearest five minutes. They also used A.M. and P.M. labels.

Current Learning

In grade 3, students tell time to the nearest minute. This is at the developmental level. They measure time intervals in minutes. They solve word problems involving addition and subtracting of time intervals in minutes using a number line or diagram. This is at the developmental level.

Routines

- The concept of estimation is a continuing routine used in this unit with the concept of time.

Future Learning

In grade 4, students will use the four operations to solve word problems involving intervals of time including problems involving simple fractions and decimals.

Additional Findings

According to *Principles and Standards for School Mathematics*, "another emphasis at this level should be on developing concepts of time and the ways it is measured. When students use calendars or sequence events in stories they are using measures of time in a real context. Opportunities arise throughout the school day for teachers to focus on time and its measurement through short conversation with their students. As teachers call attention to the clock, many young students will learn to tell time. However, this is less important than understanding patterns of minutes, hours, days, weeks, and months. (p. 104)

Children today rely heavily on digital time rather than being able to tell time on an analog clock. This is due mostly to the fact that new technology has replaced the use of analog clocks. Some math resources do not teach the skill of telling time directly.

Grade 3 Mathematics, Quarter 1, Unit 4.3

Creating and Using Picture and Bar Graphs to Solve Problems

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Represent data by drawing a scaled picture graph and scaled bar graph with several categories.
- Solve one- and two-step problems using information presented in scaled bar graphs.
- Collect measurement data by measuring lengths using rulers and show the data by making a line plot where the scale is marked off in units (whole numbers, halves, or quarters).

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the meaning of the problem.
- Analyze givens and relationships.
- Explain relationships between graphs.
- Plan a solution pathway.

Model with mathematics.

- Draw a graph to solve a problem.
- Use words to explain how to draw a scaled bar graph, scaled picture graph, and a plot graph.

Essential questions

- Based on the given data, what strategies help you make conclusions?
- What is the process for drawing a scaled bar graph?
- What is the process for drawing a scaled picture graph?
- How can you determine the operation needed to solve a problem using the data?
- How do you use a ruler to determine the length of an object to the nearest quarter inch?
- What is the process for making a line plot using measurement data?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data 3.MD

Represent and interpret data.

3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of

the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 2, students measured objects to the nearest whole unit and showed the measurements by making a line plot with a scale using whole units. Students drew a picture graph and a bar graph with single-unit scales to represent data in four categories. They solved simple one-step problems using the information presented in a bar graph.

Current Learning

In grade 3, students measure objects to the quarter inch and show the measurements by making a line plot where the scale is to the quarter inch. This is at the developmental level for halves and fourths and reinforcement for whole units. Students draw a scaled picture graph or bar graph with *several* categories. Drawing scaled picture and bar graphs is at the reinforcement level. The addition of several categories, as opposed to only three categories, is at the developmental level. Students solve one- or two-step problems using the data from the drawn graphs. Solving one-step problems is at the reinforcement level with the addition of two-step problems at the developmental level. By the end of third grade, students understand fractions as a number on a number line and represent fractions on a number line diagram.

There are no routines during this unit other than establishing classroom expectations regarding accessing and using tools and manipulatives.

Future Learning

In grade 4, students will continue to make line plots to display a data set. They will add the unit of 1/8. They will solve problems involving addition and subtraction of fractions by using the information presented in line plots.

Additional Findings

Principles and Standards of Mathematics states, "students should pose questions about themselves and their environment, issues in their school or community, and content they are studying in different subject areas" (p. 177). The book further states, "students' work with data should involve comparing related data sets. Noting the similarities and differences between two data sets requires students to become more precise in their descriptions of the data" (p. 179).

A Research Companion to Principles and Standards for School Mathematics states, "analyzing data is more than just the sum of using data-analysis techniques. It's important not to lose sight of what the data themselves have to tell us" (p. 195).

Progressions for the Common Core State Standards in Mathematics (draft) states that: "In grade 3, the most important development in data representation for categorical data is that students now draw picture graphs in which each picture represents more than one object, and they draw bar graphs in which the height of a given bar in tick marks must be multiplied by the scale factor in order to yield the number of objects in the given category. These developments connect with the emphasis on multiplication in this grade.

Grade 3 Mathematics, Quarter 4, Unit 4.4

Attributes of 2-D Shapes and Perimeter

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Identify the attributes of triangles, pentagons, hexagons, trapezoids, quadrilaterals (to include squares, rectangles, and rhombuses) and parallelograms.
- Identify how triangles, pentagons, hexagons, trapezoids, quadrilaterals (to include squares, rectangles, and rhombuses) and parallelograms share attributes.
- Identify which shapes fit into the categories of quadrilaterals and parallelograms.
- Find the perimeter of polygons when given all the side lengths.
- Find the unknown side length when given the perimeter of a polygon.
- Solve real world and mathematical problems involving perimeter.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Ability to decide when tools are appropriate and helpful.

Attend to precision.

- Define mathematical symbols and units of measure consistently and appropriately.
- Calculate accurately by rechecking for precision.
- Label for clarification.

Look for and make use of structure**.**

- Look for, develop, generalize and describe a pattern.

Essential questions

- How are the attributes/characteristics of 2-D figures of a select group/category similar and different?
- What are some ways you can classify 2-D shapes?
- What are the attributes of a given shape?
- What is the perimeter of a given figure?
- How would you use perimeter to solve a real world problem?
- Explain what perimeter is.
- When given the perimeter of a polygon with some side lengths given, how can you determine the length of an unknown side?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 3.G

Reason with shapes and their attributes.

3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Measurement and Data 3.MD

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 2, students recognized and drew shapes having specified attributes. (a given number of angles or a given number of equal faces) They identified triangles, quadrilaterals, pentagons, hexagons and cubes. They measured the length of objects using rulers, yardsticks, meter sticks and measuring tapes.

Current Learning

In grade 3, students understand that shapes in different categories (e.g. rhombuses, rectangles etc.) may share attributes (e.g., having four sides). They understand that the shared attributes can define a larger category (e.g., quadrilaterals). This is at the developmental level. They recognize rhombuses, rectangles and squares as quadrilaterals, and draw quadrilaterals that do not belong in any of these subcategories. Rhombuses are new to this grade so they are at the developmental level. The other quadrilaterals are at the reinforcement level. They solve problems finding the perimeter of polygons including problems where the side lengths are given or problems where a side length is unknown. They also can find different rectangles with the same perimeter. This is at the developmental level.

Routines: Practice multiplication and division facts to increase fluency.

Future Learning

Students in grade 4 will draw points, lines, line segments, rays, angles and perpendicular and parallel lines and identify these in two-dimensional figures. They will classify two-dimensional shapes based on parallel and perpendicular lines or size of angles. They will recognize right triangles. They will identify and draw lines of symmetry for two-dimensional figures. They will apply area and perimeter formulas for rectangles when solving problems.

Additional Findings

A Research Companion to Principles and Standards for School Mathematics states, "using manipulatives can facilitate the construction of sound representations of geometric concepts, but they must be used thoughtfully. If not, students may merely learn rote manipulation"(p. 155). The book further states that "imprecise language plagues students' work in geometry. Instruction should carefully draw distinctions between common usage and mathematical usage" (p. 154). And, "students instructed in geometry with computers often score significantly higher than those having just classroom instruction, from the elementary years to high school" (p. 156).

Principles and Standards for School Mathematics states, "the study of geometry in grades 3–5 requires thinking *and* doing. As students sort, build, draw, model, trace, measure, and construct, their capacity to visualize geometric relationships will develop. At the same time they are learning to reason, and to make, test, and justify conjectures about these relationships. This exploration requires access to a variety of tools, such as, graph paper, rulers, pattern blocks, geoboards, and geometric solids, and is greatly enhanced by electronic tools that support exploration, such as dynamic geometry software" (p. 165)